Assignment 4

Question 1:

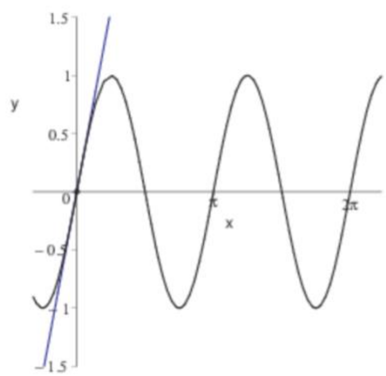
Complete the table.

$y = f(g(x))$	$u = g(x)$	$y = f(u)$
$y = (3x - 5)^3$	$u =$	$y =$

Question 2:

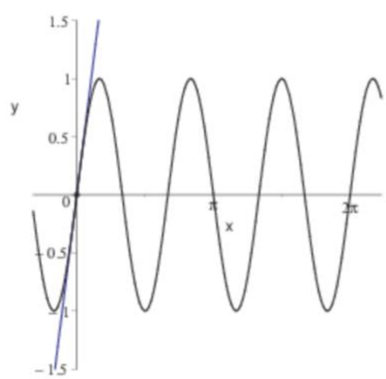
Find the slope of the tangent line to the sine function at the origin.

(a) $y = \sin(2x)$



$y'(0) =$

(b) $y = \sin(3x)$



$y'(0) =$

Compare these values with the number of complete cycles in the interval $[0,2\pi]$. What can you conclude about the slope of the sine function $\sin(ax)$ at the origin?

The slope of $\sin(ax)$ at the origin is ______ .

Question 3:

A buoy oscillates in simple harmonic motion $y = A\cos(\omega t)$ as waves move past it. The buoy moves a total of 2.4 feet (vertically) from its low point to its high point. It returns to its high point every 16 seconds.

(a) Write an equation describing the motion of the buoy if it is at its high point at $t = 0$.

$y =$ [ ]

(b) Determine the velocity of the buoy as a function of t.

$v =$ [ ]

Question 4:

Find dy/dx by implicit differentiation.

$$(\cos \pi x + \sin \pi y)^3 = 57$$

$dy/dx =$ [ ] ✖

Question 5:

Find dy/dx by implicit differentiation. and evaluate the derivate at the given point.

$$x^4 - y^6 = 0, \quad (1, 1)$$

$\frac{dy}{dx} =$ [ ]

At (1, 1): $y' =$ [ ]

Question 6:

Find the points at which the graph of the equation has a vertical or horizontal tangent line. (Order your answers from smallest to largest x, then from smallest to largest y.)

$$36x^2 + 25y^2 - 576x + 250y + 2029 = 0$$

horizontal tangents $(x, y) =$ ([ ]), ([ ])

vertical tangents $(x, y) =$ ([ ]), ([ ])

Question 7:

Use a graphing utility to sketch the intersecting graphs of the equations and determine whether or not the equations are orthogonal. [Two graphs are orthogonal if at their point(s) of intersection their tangent lines are perpendicular to each other.]

$(x - 2)^3 = 3(y - 1)$
$x(3y - 29) = 3$

STEP 1: Find the derivative of the first equation and solve for y'.

$(x - 2)^3 = 3(y - 1)$

$y' =$ ____

STEP 2: Find the derivative of the second equation and solve for y'.

$x(3y - 29) = 3$

$y' =$ ____

STEP 3: Based on your answer from Step 3, determine whether or not the two equations are orthogonal.
The two equations ---Select--- orthogonal.

Question 8:

Assume that x and y are both differentiable functions of t and find the required values of dy/dt and dx/dt.

$$y = 2\left(x^2 - 3x\right)$$

(a) Find dy/dt when $x = 2$, given that $dx/dt = 5$.
$dy/dt =$ ____
(b) Find dx/dt when $x = 6$, given that $dy/dt = 3$.
$dx/dt =$ ____

Question 9:
A point is moving along the graph of a given function such that dx/dt is 2 centimeters per second. Find dy/dt for the given values of x.

$y = 4x^2 + 2$

(a) $x = -4$
$dy/dt =$ ____ cm/sec

(b) $x = 0$
$dy/dt =$ ____ cm/sec

(c) $x = 2$
$dy/dt =$ ____ cm/sec

Question 10:

A spherical balloon is inflated with gas at a rate of 800 cubic centimeters per minute.

(a) How fast is the radius of the balloon changing at the instant the radius is 70 centimeters?

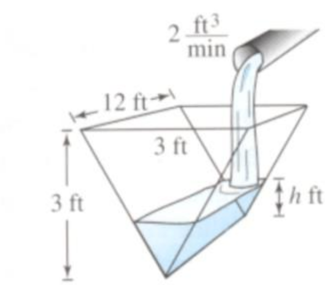
cm/min

(b) How fast is the radius of the balloon changing at the instant the radius is 90 centimeters?

cm/min

Question 11:

A trough is 12 feet long and 3 feet across the top (see figure). Its ends are isosceles triangles with altitudes of 3 feet.



(a) If water is being pumped into the trough at 2 cubic feet per minute, how fast is the water level rising when h is 1.0 feet deep?

ft/min

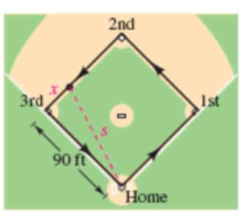
(b) If the water is rising at a rate of 3/8 inch per minute when h = 2.4, determine the rate at which water is being pumped into the trough.

ft^3/min

Question 12:

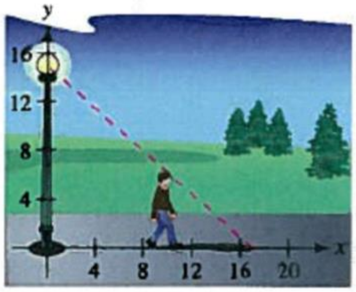
A (square) baseball diamond has sides that are 90 feet long. A player 28 feet from third base is running at a speed of 27 feet per second. At what rate is the player's distance from home plate changing? (Round your answer to two decimal places.)

ft/sec



Question 13:

A man 6 feet tall walks at a rate of 8 feet per second away from a light that is 15 feet above the ground (see figure



(a) When he is 10 feet from the base of the light, at what rate is the tip of his shadow moving?

ft/sec

(b) When he is 10 feet from the base of the light, at what rate is the length of his shadow changing?

ft/sec